particles in a block. However, when the density changes rapidly in time, the h value can be extrapolated from the previous time steps. In particular, a too rapid change in the h values may be a source of instabilities so that it is better to limit the variation to a few percent of the previous value at each time step. However, this temporal condition does not guarantee smooth spatial variations of h.

Therefore, we use a better method which suppresses the instabilities due to the variable h. This method consists in calculating the average density at the position of a particle and then taking $h^2 = K/\rho_{\text{mean}} \cdot \rho_{\text{mean}}$ is computed by averaging ρ^2 over neighbours with the SPH algorithm, i.e., according to $\rho_{\text{mean}} = (1/\rho) \sum_i m_i \rho(\mathbf{r}_i)\, w(|\mathbf{r} - \mathbf{r}_i|, h_i) = \langle \rho^2 \rangle / \rho$.

This method limits the variation of h, such that the equations of motion (2.4) are still valid in their simple form, without any inclusion of terms proportional to the derivatives of h (∇h and $\partial h/\partial t$ [6]). Evrard [4] has detailed analytically the required conditions for these added terms to be negligible, with a kernel size depending on the local density and concluded that h must vary on scales "a" much larger than h: grad h/grad $r\alpha h^2/ra \ll 1$, where r is the mean interparticle distance.

The Artificial Viscosity

For all the schemes used to simulate a fluid a crucial test lies in the shock simulation. Indeed a real shock occurs on the scale of the order of the mean free path of the fluid, which is much smaller than the resolution of the simulation. It is then necessary to smooth the density distribution over larger scales, such that the shock extends over a few resolution lengths. When applying the particle method it is convenient to have recourse to artificial viscosities that can be easily included in the equations of fluid. Using our method and our kernel we have tested several artificial viscosities proposed by Monaghan and Gingold [7] (1983) and a linear combination (CL) of the first such problems introduced:

The Neumann–Ritchmyer (NR) viscous pressure:

$$\begin{aligned} q &= \alpha \rho h^2 (\nabla \cdot \mathbf{v})^2 && \text{for } \nabla \cdot \mathbf{v} < 0 \\ &0 && \text{for } \nabla \cdot \mathbf{v} > 0 \end{aligned} \tag{4.8}$$

The Bulk (B) viscosity:

$$\begin{aligned} q &= -\alpha \rho h c (\nabla \cdot \mathbf{v}) && \text{for } \nabla \cdot \mathbf{v} < 0 \\ &0 && \text{for } \nabla \cdot \mathbf{v} > 0 \end{aligned} \tag{4.9}$$

FIG. 3. (a–f) Density and velocity profiles of an isothermal shock at the time $5C_s^{-1}$ (where C_s is the sound speed) with respectively no viscosity, Bulk viscosity ($\alpha = 0.8$), Neumann–Richtmeyer viscosity ($\alpha = 0.8$), Gingold–Monaghan viscous tensor ($\alpha_1 = 0.8$, $\alpha_2 = 0$, and $\beta = 0.1$), ($\alpha_1 = 0.8$, $\alpha_2 = 0.8$, and $\beta = 0.1$), linear combination.