Interference of radiating states and ion dynamics in spectral line broadening

I N Kosarev†∥, C Stehlé‡, N Feautrier†, A V Demura§ and V S Lisitsa§

† DAMAP, URA812, Observatoire de Paris, F-92195, Meudon Principal Cedex, France
‡ DARC, UPR176, Observatoire de Paris, F-92195, Meudon Principal Cedex, France
§ Russian Research Center 'Kurchatov Institute', 123182, Moscow, Russia

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Abstract. The influence of the plasma coupling between the populations and the coherences on the lineshape is investigated. Ion dynamics is taken into account. The present research is performed within the atomic density matrix formalism. Ion microfield dynamics is simulated by the kangaroo–Poisson stochastic process (model microfield method). Numerical calculations of both lifetimes of radiating states and lineshapes are performed for the spectral doublet (1s–2s)21S–(1s–4p)41P, (1s–2)21S–(1s–4d)41D of helium-like multicharged ions in hot dense plasmas. It is found that the ion microfield essentially influences the difference of populations of radiating 41P, 41D states. Calculation of the lineshape of the doublet (1s–2p)23P–(1s–4d)43D, (1s–2p)23P–(1s–4f)43F of neutral helium at astrophysical plasma conditions is also performed. The contribution of nonlinear interference effects (NIEF) in both allowed and forbidden components is calculated at various plasma conditions and a comparison with binary adiabatic theory is made. The results demonstrate that it is essential to take account of NIEF in the calculation of lineshapes of multicharged ions, but not essential in the case of neutral helium.

1. Introduction

The standard theory of spectral line broadening is based on the calculation of spectral functions of radiating states, while the density matrix of these states is considered to be diagonal and the calculation of populations of radiating states is considered as a separate problem in atomic kinetics. It is well known in laser physics [1] that simultaneous consideration of the dynamics of both polarizations at optical transitions and the density matrix of radiating states is important for lineshape calculations. The effects which are due to the nondiagonal matrix elements of the density matrix (coherences) are called nonlinear interference effects (NIEF) [1]. In the case of Stark-broadened lineshapes all the matrix elements of the density matrix operator (i.e. populations and coherences) are initially coupled together by the ion microfield. The optical transition takes place between these coupled states. The importance of the plasma coupling between the initial density matrix conditions and the time evolution of the optical coherences in the lineshape calculations has been demonstrated in [2] for lines of helium-like ions within the static approximation for the ion microfield, in [3] for the Lα-line of H-like ions within the molecular dynamics method

∥ Permanent address: Institute of Physics and Power Engineering, 249020, Obninsk, Russia.
for ion motions, in [4] within the binary adiabatic approximation for the low-density limit where the interaction between the radiator and the perturbing ions can be treated within the collisional theory.

The aim of this work is to investigate the influence of both NIEF and perturbing ion motion on Stark lineshapes of helium-like multicharged ions in hot dense plasmas (temperatures $T \sim 100$–1000 eV, electron densities $N_e \sim 10^{19}$–$10^{21}$ cm$^{-3}$) and their influence on lines of neutral helium in moderately dense plasmas ($T \sim 1$ eV, $N_e \sim 10^{19}$ cm$^{-3}$). These spectra are especially interesting for diagnostics of laboratory x-ray lasers (the former lines) and astrophysical stellar plasmas (the latter lines). The effects of ion motion are considered within the model microfield method (MMM) [5]. MMM is based on plasma microfield simulation by a Markov-type kangaroo–Poisson step process. The evolution operator of the radiating atom averaged over an ensemble of perturbing plasma particles is calculated exactly within MMM. The advantage of this method is that calculation of the above evolution operator is reduced to the simple calculation of the evolution operator in a static field followed by a field average. MMM enables us to investigate nonbinary dynamic effects in line broadening and has been used successfully for spectral line calculations within standard broadening theory [6–12]. Analytical calculations of metastable state lifetimes in plasma performed within MMM [13] have demonstrated the ability of MMM theory to reproduce all known analytical solutions for these lifetimes [14, 15]. This makes MMM a good approximation for the investigation of NIEF, because the correct description of radiating states kinetics is important for the calculation of spectra taking account of NIEF [2–4].

The radiating atom or ion is modelled by the three-level system shown in figure 1. The transition 3–1 is assumed to be dipole forbidden, whereas the transition 2–1 is dipole allowed (levels 2 and 3 are separated by an energy interval $\hbar\omega_{32}$). States 3 and 2 are mixed, both by electron inelastic collisions and the ion microfield. This scheme is the simplest one for investigation of NIEF [1, 2] and is relevant to the formulation of both the problem of forbidden components of spectral lines [16–19] and metastable state decay [14, 15].

![Figure 1. The scheme of levels used in the calculations.](image-url)
2. Density matrix equations. Model microfield method

The starting equations which take into account NIEF are the equations for the atomic density matrix $\rho$ (see [1–3]).

$$
\dot{\rho} = -i[\hat{H}_0 + \hat{V}(t) + \hat{\mu}(t), \rho] + \Gamma \rho + \Phi \rho + Q \tag{2.1}
$$

where $\Gamma$ is the radiative relaxation operator, $\Phi$ (assumed to be real) is the operator for collisional mixing of states by electrons, $\hat{H}_0$ is the Hamiltonian of a free atom, $Q$ is incoherent pumping (by electron impact or by recombination),

$$
\hat{V}(t) = -\mathbf{d} \cdot \mathbf{E}(t) \tag{2.2}
$$

is the interaction potential between the radiator and ion plasma microfield,

$$
\hat{\mu}(t) = \hat{G}^* e^{i\omega t} \tag{2.3}
$$

is the interaction potential of the radiator with a mode of the spontaneous radiation field which is characterized by the observed frequency $\omega$. In equation (2.1) the Poisson bracket is generalized for the case of a non-Hermitian operator, as $[O, \rho] = O\rho - \rho O^*$. In the following the carets, which denote the operators working in the Hilbert space, will be removed when particular matrix elements are considered.

Equation (2.1) is solved within perturbation theory with respect to the coupling constant $G$ (assumed to be real), i.e. $\rho = \rho^0 + \rho^n$, where $\rho^0$ determines populations and coherences of radiating states, while $\rho^n$ (for brevity denoted by $\rho$) defines polarization.

At the zeroth order of perturbation theory the following equations for $\rho^0$ are obtained [2]:

$$
\begin{align*}
\rho_{22}^0 &= -\Gamma_2 \rho_{22}^0 - 2\Phi (\rho_{22}^0 - \rho_{33}^0) - iV_{23} \rho_{32}^0 + iV_{32} \rho_{23}^0 + Q_2 \\
\rho_{33}^0 &= -\Gamma_3 \rho_{33}^0 + 2\Phi (\rho_{22}^0 - \rho_{33}^0) + iV_{23} \rho_{32}^0 - iV_{32} \rho_{23}^0 + Q_3 \\
\rho_{32}^0 &= -(\frac{1}{2} \Gamma_2 + \frac{1}{2} \Gamma_3 + 2\Phi + i\omega_{32}) \rho_{32}^0 + 2\Phi \rho_{23}^0 - iV_{32} (\rho_{22}^0 - \rho_{33}^0) \\
\rho_{31}^0 &= (\rho_{32}^0)^* \\
\end{align*} \tag{2.4}
$$

where $\Gamma_2$ and $\Gamma_3$ are radiative decay rates of states 2 and 3, respectively, $2\Phi$ is the matrix element of the electron impact broadening operator which mixes states 2 and 3.

For the polarization matrix $\hat{\rho}$, the following equations are valid at the first order in $G$:

$$
\begin{align*}
\rho_{31} &= -(i\omega_{31} + \frac{1}{2} \Gamma_2 + \frac{1}{2} \Gamma_3 + \Phi) \rho_{31} - iV_{23} \rho_{31} + i\rho_{22}^0 Ge^{-i\omega t} \\
\rho_{31} &= -(i\omega_{31} + \frac{1}{2} \Gamma_3 + \frac{1}{2} \Gamma_1 + \Phi) \rho_{31} - iV_{23} \rho_{31} + i\rho_{32}^0 G_{\text{nief}} e^{-i\omega t} \\
\end{align*} \tag{2.5}
$$

where $\omega_{ij} = \omega_i - \omega_j$ ($i, j = 1, 2, 3$) and $\Gamma_1$ is the radiative decay rate of state 1.

In this expression we note that the optical coherence $\rho_{31}$ is coupled by the radiation field to the atomic coherence $\rho_{32}$. In order to estimate the effect of this coupling (NIEF) we introduce in this equation (2.5) the constant $G_{\text{nief}}$. This constant is equal to $G$ in the exact calculation and to zero in the case where we want to check the influence of NIEF effects.

The spectrum of emitted power is defined by the work done by the probe field $G$ per unit time [11]

$$
P(\omega) = -2\omega \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} dt \text{Re}\{iG^* e^{i\omega t} \rho_{21}(t)\}. \tag{2.6}
$$

The normalized intensity for the lineshape is thus defined as

$$
I(\omega) = \frac{\text{Re}\{\int_0^{\infty} \rho_{21}(t)e^{i\omega t} dt\}}{\int_0^{\infty} \text{Re}\{\int_0^{\infty} \rho_{21}(t)e^{i\omega t} dt\}}. \tag{2.7}
$$
Let us introduce the vector $\rho(t)$ by

$$\rho(t) = (\rho_{01}(t), \rho_{02}(t), \rho_{03}(t), \rho_{04}(t), \rho_{11}(t)e^{i\omega t}, \rho_{12}(t)e^{i\omega t}, \rho_{13}(t)e^{i\omega t})^T$$

where the superscript $T$ denotes the transposed vector.

Within MMM theory, the electric field is supposed to be constant during time intervals. The jumping times follow a Poisson–kangaroo statistics with a density (i.e. frequency jump) $\nu(E)$ given by the expression [11]

$$\nu(E) = \frac{\omega_{\text{pi}}}{1 + x} \left[ \left( \frac{40\pi}{Z_i} \right)^{1/5} \left( \frac{1}{1 + Z_i} \right)^{1/10} + x \left( \frac{x + 1}{Z_i} \right) \left( \frac{1}{1 + Z_i} \right)^{1/2} + \frac{3}{2} \sqrt{\frac{\pi}{2}} \right]$$

where $Z_i$ is the charge of the perturbing ion, $\omega_{\text{pi}}$ is the ion plasma frequency, $N_e$ and $\lambda_{\text{De}}$ are the electron density and the Debye length, respectively. This choice correctly reproduces the time variation of the field autocorrelation function for the case where the field is calculated at a neutral point. It was shown that this choice is also appropriate in the case of a charged test point and low correlated plasmas [11].

The calculation of the time integral of $\rho(t)$ (equations (2.4) and (2.5)), averaged over all the realizations of the plasma microfield $E$ can be expressed as [5]

$$\{\tilde{\rho}\}_{\text{MMM}} = \left( \int_0^\infty \rho_{\text{MMM}}(t) \, dt \right) = ([T(iv)] + [vT(iv)][vI - v^2T(iv)]^{-1}[vT(iv)])Q$$

where $I$ is the unity matrix, $Q = (Q_2, Q_3, 0, 0, 0, 0)^T$ is the pumping vector, $T(iv)$ is the Laplace transform of the evolution operator $T(t)$ in the Liouville space, equal to

$$T(iv) = [vI - M]^{-1}$$

where $M = 
\begin{bmatrix}
-\Gamma_2 - 2\Phi & 2\Phi & -iV & iV \\
2\Phi & -\Gamma_3 - 2\Phi & iV & -iV \\
-iV & iV & -\frac{\Gamma_3 + \Gamma_2}{2} - 2\Phi - i\omega_{32} & 2\Phi \\
iG & -iV & 2\Phi & -\frac{\Gamma_3 + \Gamma_2}{2} - 2\Phi - i\omega_{32} \\
iG & 0 & 0 & iG_{\text{nief}} \\
0 & 0 & iG_{\text{nief}} & 0 \\
0 & 0 & 0 & 0 \\
-\frac{\Gamma_3 + \Gamma_2}{2} - \Phi + i\Delta\omega_{21} & -iV & 0 & 0 \\
-iV & -\frac{\Gamma_3 + \Gamma_2}{2} - \Phi + i\Delta\omega_{31} & 0 & 0
\end{bmatrix}

(2.10)

where $\Delta\omega_{ij} = \omega - \omega_{ij}, V = \alpha E, \alpha = \langle 3|d|2 \rangle$.

In the right-hand side of equation (2.9), the brackets denote the average over the static field distribution function $P(E)$, for example,

$$[T(iv)] = \int_0^\infty dE \, P(E)T(iv).$$
Expanding (2.9) and (2.10) in powers of $G$ and keeping the terms linear with respect to $G$ (which contribute to the emitted power) gives the following expression for the normalized lineshape:

$$I(\omega) = \frac{\text{Im}[(0, 0, 0, 1, 0)(T^{(1)})_{\text{MMM}}Q]}{\pi \{\rho^0\}_{22,\text{MMM}}}$$  \hspace{1cm} (2.11)

with

$$(T^{(1)})_{\text{MMM}} = \{\Delta T\} + \{\nu T'\} \{\nu I - \nu^2 T'\}^{-1} \{\nu^2 T'\}$$  \hspace{1cm} (2.12)

$$T' = \begin{bmatrix} T^0 & 0 \\ 0 & T^\mu \end{bmatrix}$$

$$\Delta T = T'GT'.$$

In this equation $T^0$ is the Fourier transform (at zero frequency) of the static evolution operator for 'populations' (2.4) and $T^\mu$ is the Fourier transform (at zero frequency) of the static evolution operator for polarizations (2.5). $T'$ has dimension $6 \times 6$ whereas $T^0$ is $4 \times 4$ and $T^\mu$ is $2 \times 2$. $G$ is the matrix obtained from $M$ (2.10) when all the terms are equal to zero except $G$ and $G_{\text{nef}}$. Note that $\{\rho^0\}_{\text{MMM}}$ can be calculated by the same expression as (2.9) for $\{\tilde{\rho}(t)\}_{\text{MMM}}$ substituting $T^0$ for $T$.

The static limit is obtained by putting $\nu = 0$ in equation (2.12). Only the first term of (2.12) contributes in this limit.

The expressions (2.11) and (2.12) for the normalized lineshape contain the effects of both ion motion and NIEF. To investigate the influence of NIEF we will compare this total lineshape with the standard one which is equal to [5]

$$I_{\text{standard}}(\omega) = \frac{1}{\pi} \text{Re}(\{T^\mu(\omega)\}_{\text{MMM}})_{11}$$  \hspace{1cm} (2.13)

$$(T^\mu(\omega))_{\text{MMM}} = \{T^\mu(\omega)\} + \{\nu T^\mu(\omega)\} \{\nu I - \nu^2 T^\mu(\omega)\}^{-1} \{\nu^2 T^\mu(\omega)\}.$$

3. Numerical calculations of metastable state decay and lineshapes

Numerical calculations for lineshapes require the knowledge of the field distribution function $P(E)$. In the case of multicharged helium-like ions, this quantity has been obtained numerically, either with the Baranger–Mozer formulation (at low densities) or with Monte Carlo simulation (at high densities) [20]. For simplicity we consider an 'ideal' plasma, composed of electrons and He-like ions with total charge $Z_i$. In the case of neutral helium the Holtsmark function was used.

For multicharged ions, all necessary atomic constants which define radiative and electron collisional decay rates and the interaction with the plasma ion microfield are taken from [2]. Hereafter we use a simplified three-level model, neglecting the degeneracy over the magnetic quantum numbers. This degeneracy is included approximately by the use of averaged atomic constants.

3.1. Metastable state decay

The lifetime is defined as the time integral of the diagonal matrix elements of the density matrix, i.e. $\tau_i = \int_0^\infty \rho^0_i(t) \, dt = \tilde{\rho}^0_i$ for level $i$.

MMM calculations have been performed for the populations of $4^1P, 4^1D$ states of the helium-like ion Al$^{11+}$ at a plasma temperature of $T = 350$ eV. The former state corresponds
Table 1. Populations of metastable state (normalized to $\tilde{\rho}_{22}^0$).

<table>
<thead>
<tr>
<th>$N_e$ (cm$^{-3}$)</th>
<th>MMM, this work</th>
<th>Radiative-collisional model</th>
<th>MMM [13]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10$^{20}$</td>
<td>1.27</td>
<td>1.39</td>
<td>1.30</td>
</tr>
<tr>
<td>10$^{19}$</td>
<td>3.45</td>
<td>4.94</td>
<td>4.01</td>
</tr>
<tr>
<td>10$^{18}$</td>
<td>22.7</td>
<td>40.4</td>
<td>31.1</td>
</tr>
<tr>
<td>10$^{17}$</td>
<td>199.0</td>
<td>395.0</td>
<td>302.0</td>
</tr>
</tbody>
</table>

The latter state is state 3. The decay rate of state 2 equals $\Gamma_2 = 4.1 \times 10^{12}$ s$^{-1}$, whereas the radiative decay rate of state 3 is negligible. Estimation of the electron collisional decay rate $2\Phi$ gives

$$2\Phi = 3.1 \times 10^{-3} N_e (Z_i - 1)^{-2} T^{-1/2}$$  \hspace{1cm} (3.1)

where $2\Phi$ is expressed in Hz, $N_e$ in cm$^{-3}$, temperature $T$ in eV. For the field $E$, created by an Al$^{11+}$ ion at the mean interionic distance, the average over magnetic quantum numbers of the matrix elements $V_{23}$ and $V_{32}$ of $V$ (equation (2.2)) is approximately equal to

$$V = 30 (N_i)^{2/3}$$  \hspace{1cm} (3.2)

where $V$ is expressed in angular frequency units, $N_i$ is the ionic density in cm$^{-3}$. According to the formulation of the problem of metastable state decay the pumping exists only in this state. Numerical MMM calculations of the ratio $\tilde{\rho}_{33}^0 / \tilde{\rho}_{22}^0$ are presented in the second column of table 1. The third column contains the results for this ratio calculated within the radiative-collisional model which takes into account only radiative and electron-impact relaxation. In the fourth column, we report the values given by the following formula:

$$\tilde{\rho}_{33}^0 / \tilde{\rho}_{22}^0 = 1 / (2\Phi / \Gamma_2 + v_i^{ad} / \Gamma_2)$$

$$v_i^{ad} = 18.2 \times \Gamma_2 N_i \left( \frac{\alpha}{\omega_{32}} \right)^{3/2}$$  \hspace{1cm} (3.3)

In this formula the influence of the ion microfield is taken into account by an analytical expression for the adiabatic decay rate $v_i^{ad}$ [14, 15] calculated in [13] within a simplified version of MMM which does not account for Debye screening. The domain of validity of this formula is $v_i N_i^{1/3} / \Gamma_2 \gg 1$, $\alpha N_i^{2/3} / \omega_{32} \\ \approx 1$.

As one can see from this table, the ion microfield essentially changes the difference of populations of radiating states. Namely, this quantity defines the influence of NIEF on lineshape (see [2, 4] and the next paragraph). The discrepancy between numerical and analytical results at low densities is due to the breakdown of the adiabatic decay rate formula (the parameter $v_i N_i^{1/3} / \Gamma_2$ is approximately equal to unity at $N_e = 10^{17}$ cm$^{-3}$).

3.2. Lineshapes of the spectral doublet $2^1 S$–$4^1 P, 2^1 S$–$4^1 D$ of the helium-like ion Al$^{11+}$

Here the relevant atomic parameters are

$$\Gamma_1 = \Gamma = 0, \hspace{1cm} \Gamma_2 = 4.1 \times 10^{12} \text{ s}^{-1}, \hspace{1cm} \omega_{32} = -4.1 \times 10^{14} \text{ s}^{-1}$$

At $T = 350$ eV and for an electron density $N_e = 10^{21}$ cm$^{-3}$ we note a good agreement between static and dynamic lineshapes (see figure 2). The NIEF contribution is negligible both for the allowed and forbidden components. This NIEF contribution corresponds to the difference between the line intensity calculated with $G_{nief} = G$ (exact lineshape) and $G_{nief} = 0$. 
Interference and ion dynamics effects in line broadening

As one can see from figures 3(a) and (b) at plasma temperature $T = 350$ eV and electron density $N_e = 5 \times 10^{19}$ cm$^{-3}$, the influence of plasma ion motion is important for both the allowed and the forbidden components of the lineshape. The influence of NIEF on the lineshape at these plasma conditions is presented in figures 3(c) and (d). Taking account of this effect the maximum of the forbidden component is increased by approximately 30%. The influence of NIEF on the allowed component is not essential (about 5%). The comparison of static and dynamic NIEF contributions indicates that the ion dynamics effects decrease this contribution (see figure 3(e)). Taking account of the Doppler effect, which is essential with these plasma conditions, does not change the NIEF influence considerably (see figure 3(f) where the convolution of Stark and Doppler lineshapes is presented).

At $T = 100$ eV the effects of ion motion are essential for the allowed component (see figure 4(a)) and less important for the forbidden component (see figure 4(b), the difference between static and dynamic lineshapes is about 10%). A comparison of figures 3(b), 4(b) and (d) shows that NIEF increases with temperature. This tendency is conserved taking account of the Doppler effect (figures 3(f), 4(c), (e)).

It follows from figures 5(a)–(d) that the influence of NIEF is increased with the relative pumping rate on metastable state 3, this is still valid after taking account of the Doppler effect.

The plasma conditions considered in this section (except for the case $N_e = 10^{21}$ cm$^{-3}$) correspond to the case $g \sim 1$, where

$$g = N_i \rho_W^3, \quad \rho_W = \left(\frac{\pi a^2}{2 \omega_{21} v_i}\right)^{1/3}$$ (3.4)
Figure 3. Lineshape of the doublet $2^1S\rightarrow4^1P, 2^1S\rightarrow4^1D$ of Al$^{11+}$, $T = 350$ eV, $N_e = 5 \times 10^{19}$ cm$^{-3}$, $Q_2 = Q_3$. (a) Full curve, total lineshape; broken curve, total static lineshape. (b) The same as in (a) but only the forbidden component is plotted. (c) Full curve, total lineshape; long-broken curve, standard lineshape; short-broken curve, NIEF contribution. (d) The same as in (c) but only the forbidden component is plotted. (e) Full curve, static NIEF contribution; broken curve, dynamic NIEF contribution. (f) Convolution of Doppler and MMM Stark lineshapes. Full curve, total lineshape; broken curve, standard lineshape.
where $\rho_W$ is the Weisskopf radius for quadratic Stark effect, $v_i$ being the mean ion velocity. The domain of validity of the binary theory in the centre of the line components is $g \ll 1$. 

*Figure 3. Continued.*
According to the binary adiabatic theory result [4], generalized to include the decay rate due to ion collisions, the ratio of the maximum intensity of the NIEF contribution and
Figure 4. Lineshape of the doublet $2^1\text{S}$$\rightarrow$$4^1\text{P}$, $2^1\text{S}$$\rightarrow$$4^1\text{D}$ of Al$^{11+}$. $N_e = 5 \times 10^{19}$ cm$^{-3}$, $Q_2 = Q_3$.

(a) $T = 100$ eV. Full curve, total lineshape; short-broken curve, standard lineshape; long-broken curve, NIEF contribution; short–long-broken curve, total static lineshape. (b) The same as in (a) but only the forbidden component is plotted. (c) Convolution of Doppler and Stark MMM lineshapes, $T = 100$ eV (forbidden component). Full curve, total lineshape; broken curve, standard lineshape. (d) $T = 500$ eV (forbidden component). Full curve, total lineshape; broken curve, NIEF contribution; chain curve, standard lineshape. (e) Convolution of Doppler and Stark MMM lineshapes, $T = 500$ eV (forbidden component). Full curve, total lineshape; broken curve, standard lineshape.
the maximum intensity of standard forbidden components is $I_{\text{nief}}/I_{\text{forb}} = (\tilde{\rho}_{33}^0 - \tilde{\rho}_{22}^0)/\tilde{\rho}_{22}^0$. The comparison of these values is presented in table 2 for different plasma conditions.
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This illustrates that the NIEF contribution follows the mean differences of populations of radiating states.

Previous adiabatic calculations have shown that the striking manifestation of NIEF is the possibility of an abrupt increase of intensity in the wings of the allowed and forbidden lines at low densities [4]. One can see from figures 6(a) and (b) \((T = 350 \text{ eV and } N_e = 10^{18} \text{ cm}^{-3})\) that this effect is reproduced by MMM. Results of MMM calculations are in qualitative agreement with binary adiabatic theory in the nearest wings.

3.3. Lineshapes of the spectral doublet \(2^1S-4^1P, 2^1S-4^1D\) of the helium-like ion \(Ni^{26+}\)

The relevant parameters here are

\[
\Gamma_3 = \Gamma_1 = 0 \quad \Gamma_2 = 8.3 \times 10^{13} \text{ s}^{-1} \quad \omega_{32} = 1.3 \times 10^{15} \text{ s}^{-1}.
\]
Figure 5. Lineshape of the doublet $2^1S-4^1P, 2^1S-4^1D$ of $A^1_{i+}, T = 350$ eV, $N_e = 5 \times 10^{19}$ cm$^{-3}$ (forbidden components). (a) $Q_2/Q_3 = 0.5$. Full curve, total lineshape; broken curve, standard lineshape; dotted curve, NIEF contribution. (b) Convolution of Doppler and Stark MMM lineshapes, $Q_2/Q_3 = 0.5$. Full curve, total lineshape, broken curve, standard lineshape. (c) $Q_2/Q_3 = 2$. Full curve, total lineshape, broken curve, standard lineshape; dotted curve, NIEF contribution. (d) Convolution of Doppler and Stark MMM lineshapes, $Q_2/Q_3 = 2$. Full curve, total lineshape; broken curve, standard lineshape.
For a plasma temperature of $T = 1000$ eV and an electron density of $N_e = 10^{21}$ cm$^{-3}$ ($g \sim 1$) the ion motion effects are important for the lineshape (see Figure 5. Continued.)
Figure 6. Lineshape of the doublet $^2\text{S}^{}$–$^4\text{P}^{}$, $^2\text{S}^{}$–$^4\text{D}^{}$ of Al$^{11+}$, $T = 350$ eV, $N_e = 10^{18}$ cm$^{-3}$, $Q_2 = Q_3$. (a) Full thin curve, total MMM lineshape; full thick curve, total theoretical (according to [4]) lineshape; broken thin curve, standard lineshape (MMM), broken thick curve, standard lineshape (theoretical). (b) Convolution of Doppler and Stark MMM lineshapes. Full curve, total lineshape; broken curve, standard lineshape.
Figure 7. Lineshape of the doublet $^2S-^4P, ^2S-^4D$ of Ni$^{26+}, T = 1000$ eV, $N_e = 10^{21}$ cm$^{-3}$, $Q_2 = Q_3$. (a) Thin curve, total static lineshape; thick curve, total lineshape. (b) Full curve, total lineshape; broken curve, standard lineshape; chain curve, NIEF contribution. (c) Full curve, NIEF contribution; broken curve, static NIEF contribution. (d) Convolution of Doppler and MMM Stark lineshapes. Full curve, total lineshape; broken curve, standard lineshape.
Figure 7. Continued.

Figure 7(a)). NIEF increases the intensity of the forbidden component by approximately a factor of two and decreases the intensity of the allowed component by 15%.
Figure 8. Lineshape of the doublet $2^1S\rightarrow4^1P$, $2^1S\rightarrow4^1D$ of Ni$^{26+}$, $T = 1000$ eV, $N_e = 10^{20}$ cm$^{-3}$, $Q_2 = Q_3$. (a) Full curve, total lineshape; short-broken curve, standard lineshape; long-broken curve, NIEF contribution. (b) The same as in (a) but only the forbidden component is plotted. (c) Convolution of Doppler and MMM Stark lineshapes. Full curve, total lineshape; broken curve, standard lineshape.
The dynamical NIEF contribution is slightly smaller than the static one (figure 7(c)). The Doppler broadening partially smoothes the gap between the allowed and forbidden components but does not change the picture of NIEF influence considerably.

At $T = 1000$ eV and $N_e = 10^{20}$ cm$^{-3}$, the comparison between the standard and total MMM profiles indicates that the influence of NIEF is increased and the noticeable forbidden component is only due to NIEF (figures 8(a) and (b)). The intensity of the allowed component is decreased by approximately 20%. The Doppler effect (figure 8(c)) totally smoothes the gap between the allowed and forbidden components and, taking account of NIEF, gives an important shoulder near the allowed component. At these plasma conditions $g \sim 0.1$, the binary theory is applicable. There is a good agreement (within 20%) with binary adiabatic theory for maximum intensities of both allowed and forbidden components.

As one can see from table 3, the ratio of the maximum values of the NIEF contribution to the intensity of the standard forbidden component follows the differences of the populations of radiating states.

<table>
<thead>
<tr>
<th>$N_e$ (cm$^{-3}$)</th>
<th>$(\tilde{\rho}<em>{33}^0 - \tilde{\rho}</em>{22}^0)/\tilde{\rho}_{22}^0$</th>
<th>$I_{nief}(\text{max})/I_{forb}(\text{max})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{21}$</td>
<td>1.59</td>
<td>1.84</td>
</tr>
<tr>
<td>$10^{20}$</td>
<td>15</td>
<td>14.9</td>
</tr>
</tbody>
</table>
3.4. Lineshape of the $^2\text{P}-^4\text{D}$, $^2\text{P}-^4\text{F}$ spectral doublet of neutral helium

In the case of neutral helium state $^2\text{P}$ corresponds to state 1 of the model, $^4\text{D}$ to state 2 and $^4\text{F}$ to state 3. Radiative decay rates of these states are of the same order of magnitude ($\Gamma_1 = 0.1 \times 10^8 \text{ s}^{-1}$, $\Gamma_2 = 0.32 \times 10^8 \text{ s}^{-1}$, $\Gamma_3 = 0.14 \times 10^8 \text{ s}^{-1}$, $\omega_{32} = -1.36 \times 10^{12} \text{ s}^{-1}$ [21]). The decay rate of electron collisional mixing of states 2 and 3 is estimated by [19]

$$2\Phi = 4\pi N_e \frac{e^2}{\hbar^2} \frac{\alpha^2}{v_e} \ln \left( \frac{\lambda_{\text{D}}}{\rho_{\text{W}}} \right)$$ (3.5)

where $v_e$ is the mean electron velocity. Calculations of the lineshape of helium in a hydrogen plasma at an electron density of $N_e = 10^{15} \text{ cm}^{-3}$ and $T = 10^4 \text{ K}$ show that taking account of NIEF is not essential either for the allowed or for the forbidden components (figure 9). The maximum values of allowed and forbidden components are in qualitative agreement with those obtained by the binary [4] and the unified theories [18] (at these conditions $g \sim 0.1$). The negligible influence of NIEF at this plasma condition is explained by the large value of the electron collisional rate which is three orders of magnitude larger than the radiative decay rates [4]. The present negligible NIEF effects for helium lines justify the use of the MMM standard theory in the calculations performed by Schöning [12] at different plasma conditions.

![Figure 9. Lineshape of the doublet $^2\text{P}-^4\text{D}$, $^2\text{P}-^4\text{F}$ of He, $T = 10^4 \text{ K}$, $N_e = 10^{15} \text{ cm}^{-3}$. Full curve, total lineshape; thick broken curve, standard lineshape; thin broken curve, NIEF contribution.](image)

4. Conclusions

Taking account of the ion microfield is essential for atomic state kinetics of multicharged ions in hot dense plasmas. The difference of the populations of radiating states calculated...
taking into account the decay rates due to interactions with the plasma ions differs strongly from calculations within the radiative–collisional model.

Taking account of both ion motion and NIEF is essential for calculations of lineshapes of helium-like multicharged ions in hot dense plasmas at moderate densities. The influence of NIEF follows the difference of mean populations of radiating states and increases with the plasma temperature, the pumping rate on the metastable state, the charge of the radiating ion, with the decrease of plasma density. MMM theory reproduces well the sharp increase of intensity in the nearest wings of the allowed and the forbidden components at low plasma density, as expected from earlier work [4]. At large plasma densities there is good agreement between static and dynamical lineshapes, for the cases investigated in this paper. It is not essential to take account of NIEF in the calculations of the lineshapes of neutral helium in plasmas of moderate densities.

These calculations have been performed using a simple three-level system, neglecting the degeneracy of the magnetic quantum numbers. This approximation prevents us from correctly performing the average over all the directions of the plasma microfield (angular average). Thus the present results must be taken as a first step, showing the trends with the plasma conditions and the nature of the radiating element.

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